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Question Paper Code : 31263

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Second Semester

Civil Engineering

MA 2161 — MATHEMATICS — II

(Common to all branches)

(Regulation 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the particular integral of $\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2+3x+6$.

2. Show that e^{-x}, xe^{-x} are independent solutions of $y''+2y'+y=0$ in any interval.

3. What is the directional derivative of $\phi = x^2yz-14xz^2$ at the point $(1,-2,-1)$ in the direction of the vector $2\hat{i}-\hat{j}-2\hat{k}$?

4. If S is any closed surface enclosing a volume V and $\overline{F}=ax\hat{i}+by\hat{j}+cz\hat{k}$, find $\int_S \overline{F}\cdot d\overline{S}$.

5. Show that $f(z)=z+2\overline{z}$ is not analytic anywhere in the complex plane.

6. Find the invariant points of the transformation $w=\frac{z-1}{z+1}$.

7. Evaluate $\oint_C \frac{z+2}{z}dz$ where C is the semi-circle $|z|=2$ in the upper half of the z-plane.

8. Identify the singularities of $f(z)=\frac{z^2}{(z-3)^2(z^2+16)}$.

9. Find $L(f(t))$ if $f(t) = \begin{cases} \cos(t - 2\pi/3), & t > 2\pi/3 \\ 0, & t < 2\pi/3. \end{cases}$

10. Find the inverse Laplace transform of $\dfrac{6s}{s^2 - 16}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve by the method of variation of parameters :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x. \quad (8)$$

(ii) Solve : $Dx + Dy + 3x = \sin t$

$$Dx + y - x = \cos t. \quad (8)$$

Or

(b) (i) Solve : $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = \log x \sin(\log x)$. (8)

(ii) Solve by the method of undetermined coefficients :

$$(D^2 - 2D)y = e^x \sin x. \quad (8)$$

12. (a) (i) Verify Green's theorem for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region bounded by $x = 0$, $y = 0$ and $x + y = 1$. (8)

(ii) Verify Gauss divergence theorem for $\overline{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ where S is the surface of the cuboid formed by the planes $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$ and $z = c$. (8)

Or

(b) (i) A fluid motion is given by $V = ax\hat{i} + ay\hat{j} - 2az\hat{k}$. Is it possible to find out the velocity potential? If so, find it. Is the motion possible for an incompressible fluid? (8)

(ii) Verify Stoke's theorem for $\overline{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary in the xy-plane. (8)

13. (a) (i) If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f(z)|=0$. (8)

(ii) Find the analytic function $z=u+iv$, if $u-v=\frac{x-y}{x^2+4xy+y^2}$. (8)

Or

(b) (i) Show that the transformation $w=\frac{1}{z}$ transforms all circles and straight lines into the circles and straight lines in the w-plane. Which circles in the z-plane become straight lines in the w-plane and which straight lines are transformed into other straight lines? (8)

(ii) Discuss the transformation $w=\frac{i(1-z)}{1+z}$ and show that it maps the circle $|z|=1$ into the real axis of the w-plane and the interior of the circle $|z|<1$ into the upper half of the w-plane. (8)

14. (a) (i) Find all possible Laurent's expansions of the function $f(z)=\frac{4-3z}{z(1-z)(2-z)}$ about $z=0$. Indicate the region of convergence in each case. Find also the residue of $f(z)$ at $z=0$, using the appropriate Laurent's series. (10)

(ii) Evaluate $\int_C \frac{z\,dz}{(z-1)(z-2)^2}$, where C is the circle $|z-2|=\frac{1}{2}$, using Cauchy's residue theorem. (6)

Or

(b) (i) Evaluate $\int_{-\infty}^{\infty}\frac{x^2dx}{(x^2+a^2)(x^2+b^2)}$ using contour integration, with $a>b>0$. (8)

(ii) Using Cauchy's integral formula, evaluate $\int_C \frac{z+1}{z^3-2z^2}dz$ where C is the circle $|z-2-i|=2$. (8)

15. (a) (i) Find $L^{-1}\left(\frac{s}{(s^2+1)^2}\right)$. Hence find $L^{-1}\left(\frac{1}{(s^2+1)^2}\right)$. (6)

(ii) Using convolution theorem, evaluate $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$. (5)

(iii) Find the Laplace transform of the function $f(t)=\begin{cases} a\sin wt, & 0\le t\le \frac{\pi}{w} \\ 0, & \frac{\pi}{w}\le t\le \frac{2\pi}{w} \end{cases}$. (5)

Or

(b) (i) Solve $y''-4y'+8y=e^{2t}$, $y(0)=2$ and $y'(0)=-2$ using Laplace transform. (8)

(ii) Verify the initial and final value theorems when $f(t)=L^{-1}\left(\frac{1}{s(s+2)^2}\right)$. (8)